

## Math 42 Midterm 2 Review

You will need a calculator to solve the problems marked ★.

You should **NOT** use a calculator for any other problems.

- [0] Print out the inverse trigonometric values “flashcard” from my website, and cut into squares. Randomly select an inverse trigonometric function “flashcard” and a trigonometric value “flashcard”. Identify the value of the inverse trigonometric function as applied to the trigonometric value.

**Some combinations will generate values that can only be found using a calculator, while other combinations will not generate any value. You should be able to identify those.**

Repeat as many times as necessary to master the inverse trigonometric function values.

- [1] Identify the middle value, amplitude, period and phase shift. Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete cycles, starting at the phase shift. Sketch a detailed graph of 2 complete cycles using the information you found. Label all  $x$  – and  $y$  – values for the 9 points on the appropriate axes, using a consistent scale for each axis. Also, write the equation and sketch the graph of the corresponding reciprocal function.

[a]  $y = 2 \sin(\frac{1}{2}x - \frac{5\pi}{4}) - 3$

[b]  $y = -3 \cos(\frac{3\pi}{4}x - \frac{\pi}{2}) - 4$

[c]  $y = -5 \sin(\frac{5\pi}{3}x + \frac{2\pi}{9}) + 1$

[d]  $y = 4 \cos(3x + \frac{5\pi}{3}) + 2$

- [2] Sketch the graphs. **You only need to get the general position and shape correct. Do NOT plot points.** Find the domain, range, and equations of all asymptotes.

[a]  $y = \sec x$

[b]  $y = \csc x$

[c]  $y = \cot x$

[d]  $y = \tan x$

[e]  $y = \sin^{-1} x$

[f]  $y = \cos^{-1} x$

[g]  $y = \tan^{-1} x$

- [3] Fill in the blanks. Use the graphs from [2].

[a] As  $x \rightarrow -\frac{\pi}{2}^+$ ,  $\sec x \rightarrow$  \_\_\_\_\_ and  $\tan x \rightarrow$  \_\_\_\_\_

[b] As  $x \rightarrow 1^-$ ,  $\cos^{-1} x \rightarrow$  \_\_\_\_\_

[c] As  $x \rightarrow -\pi^+$ ,  $\csc x \rightarrow$  \_\_\_\_\_ and  $\cot x \rightarrow$  \_\_\_\_\_

[d] As  $x \rightarrow \infty$ ,  $\tan^{-1} x \rightarrow$  \_\_\_\_\_

[e] As  $x \rightarrow \frac{3\pi}{2}^-$ ,  $\sec x \rightarrow$  \_\_\_\_\_ and  $\tan x \rightarrow$  \_\_\_\_\_

[f] As  $x \rightarrow -1^+$ ,  $\sin^{-1} x \rightarrow$  \_\_\_\_\_

[g] As  $x \rightarrow 0^-$ ,  $\csc x \rightarrow$  \_\_\_\_\_ and  $\cot x \rightarrow$  \_\_\_\_\_

[h] As  $x \rightarrow -\frac{3\pi}{2}^+$ ,  $\sec x \rightarrow$  \_\_\_\_\_ and  $\tan x \rightarrow$  \_\_\_\_\_

[i] As  $x \rightarrow -1^+$ ,  $\cos^{-1} x \rightarrow$  \_\_\_\_\_

[j] As  $x \rightarrow 0^+$ ,  $\csc x \rightarrow$  \_\_\_\_\_ and  $\cot x \rightarrow$  \_\_\_\_\_

[k] As  $x \rightarrow -\infty$ ,  $\tan^{-1} x \rightarrow$  \_\_\_\_\_

[l] As  $x \rightarrow \frac{\pi}{2}^-$ ,  $\sec x \rightarrow$  \_\_\_\_\_ and  $\tan x \rightarrow$  \_\_\_\_\_

[m] As  $x \rightarrow 1^-$ ,  $\sin^{-1} x \rightarrow$  \_\_\_\_\_

[h] As  $x \rightarrow \pi^-$ ,  $\csc x \rightarrow$  \_\_\_\_\_ and  $\cot x \rightarrow$  \_\_\_\_\_

[4] Simplify the following expressions. Some expressions have no value.

- |                                      |                                      |                                       |  |
|--------------------------------------|--------------------------------------|---------------------------------------|--|
| [a] $\sin(\sin^{-1} \frac{2}{3})$    | [b] $\tan^{-1}(\tan \frac{\pi}{6})$  | [c] $\cos^{-1}(\cos(-\frac{\pi}{3}))$ | [d] $\sin^{-1}(\sin(-\frac{\pi}{4}))$  |
| [e] $\cos^{-1}(\cos \frac{5\pi}{6})$ | [f] $\sin^{-1}(\sin \frac{2\pi}{3})$ | [g] $\tan(\tan^{-1}(-\frac{5}{4}))$   | [h] $\cos(\cos^{-1} \frac{4}{3})$      |
| [i] $\tan(\tan^{-1} \frac{4}{5})$    | [j] $\cos(\cos^{-1}(-\frac{3}{4}))$  | [k] $\sin(\sin^{-1}(-\frac{3}{2}))$   | [l] $\tan^{-1}(\tan(-\frac{3\pi}{4}))$ |

[5] Simplify the following expressions. Some expressions have no value.

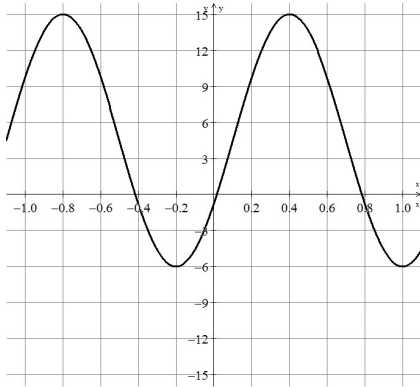
- |                                   |                                     |                                   |                                     |
|-----------------------------------|-------------------------------------|-----------------------------------|-------------------------------------|
| [a] $\sin(\tan^{-1} \frac{2}{3})$ | [b] $\sec(\sin^{-1}(-\frac{3}{4}))$ | [c] $\csc(\cos^{-1} \frac{3}{2})$ | [d] $\cos(\sin^{-1} \frac{4}{5})$   |
| [e] $\tan(\cos^{-1} \frac{1}{3})$ | [f] $\cot(\sin^{-1}(-\frac{4}{3}))$ | [g] $\csc(\tan^{-1}(-2))$         | [c] $\cot(\cos^{-1}(-\frac{2}{5}))$ |

[6] Simplify the following expressions.

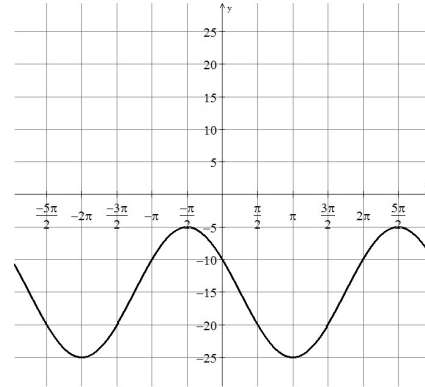
- |                            |                                   |  |
|----------------------------|-----------------------------------|--|
| [a] $\tan(\sin^{-1}(x+1))$ | [b] $\cos(\tan^{-1} \frac{t}{2})$ | [c] $\sin(\cos^{-1} \frac{\sqrt{1-y}}{2})$ |
|----------------------------|-----------------------------------|--|

[7] Find an equation for each graph.

[a]



[b]



[8] AJ has been reading about biorhythms, and decided to make a chart of his overall mood starting on Jan 1. On a scale of 1 to 10, AJ's mood reached a high of 9.5 on Jan 17, and dropped continually to a low of 2 on Feb 2, before starting to rise again. Assume AJ's mood corresponds to a sinusoidal function.

- [a] Find an equation for his mood on the  $t^{th}$  day since Jan 1.  
 [b] ★ What will AJ's mood be on Mar 19?

[9] ★ A 265 meter cable connects the roofs of two buildings.

From the roof of the first building, the angle of depression to the roof of the second building is  $41^\circ$ .

From the base of the first building, the angle of elevation to the roof of the second building is  $67^\circ$ .

- [a] Find the height of each building, and the distance between the bases of the buildings.  
 [b] Find the angle of elevation of the roof of the first building from the base of the second building.

[10] ★ A 17 foot ladder is leaning against the wall of a building. The base of the ladder is 8 feet from the base of the building.

- [a] Find the angle between the ladder and the building.  
 [b] Find the angle between the ladder and the ground.

[11] If a ferris wheel is turning at a constant rate, then the height of a particular seat relative to the center of the wheel is a form of simple harmonic motion.

Consider a ferris wheel of radius 65 feet, which takes 8 seconds for a seat to go from the bottom of the wheel to the top. Assume the height of a seat is considered positive if it is above the center of the ferris wheel, and negative if it is below.

- [a] What is the frequency of the wheel ?
- [b] Find the equation of motion of a seat which starts at the bottom of the wheel.
- [c] Find the equation of motion of a seat which is directly to the right of the center of the wheel, if the wheel is turning clockwise.

[12] ★ A car is travelling at 57 miles per hour directly southward.

At 2pm, the car is 81 miles from the center of a town, on a bearing of  $281^\circ$  from the center.

At 2:30pm, the car turns and begins travelling directly eastward at the same speed.

- [a] At 2pm, how far is the car north/south and east/west of the center of town ?
- [b] At 2:30pm, how far is the car north/south and east/west of the center of town ?
- [c] At 2:30pm, how far is the car from the center of town, and on what bearing from the center ?
- [c] At what time will the car be directly southeast of the center of town ?